



BOARD OF STUDIES
NEW SOUTH WALES

2002 HSC Mathematics Extension 2 Marking Guidelines

Question 1 (a)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Correctly evaluates integral, possibly omitting the constant of integration	2
• Obtains a relation equivalent to $du = \sec x \tan x dx$ OR • Obtains the expression $\int u^2 du$	1

Question 1 (b)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correctly evaluates integral by completing the square, possibly omitting the constant of integration	2
• Completes the square correctly (ie $x^2 + 2x + 2 = (x+1)^2 + 1$)	1

Question 1 (c)
Outcomes assessed: E8
MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Correctly evaluates integral, possibly omitting the constant of integration Obtains result $\frac{3}{4}\ln(x+3) + \frac{1}{4}\ln(x-1) + c$ 	3
<ul style="list-style-type: none"> Obtains partial fraction expression $\frac{3}{4}\left(\frac{1}{x+3}\right) + \frac{1}{4}\left(\frac{1}{x-1}\right)$ 	2
<ul style="list-style-type: none"> Gives partial fraction format $\frac{A}{x+3} + \frac{B}{x-1}$ 	1

Question 1 (d)
Outcomes assessed: E8
MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Obtains correct answer 	4
<ul style="list-style-type: none"> Correctly integrates by parts twice, using correct choice for u, dv but fails to obtain final result 	3
<ul style="list-style-type: none"> Correctly integrates by parts once, handling limits correctly 	2
<ul style="list-style-type: none"> Integrates by parts once. Gets correct result for integral except for a change of sign or failure to handle limits of integration 	1

Question 1 (e)
Outcomes assessed: HE6
MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Obtains correct result 	4
<ul style="list-style-type: none"> Obtains correct primitive and correct limits of integration 	3
<ul style="list-style-type: none"> Obtains expression $\int_0^1 \frac{2dt}{t^3+3}$ or equivalent (in terms of t) or obtains correct primitive but with incorrect (or no) limits of integration 	2
<ul style="list-style-type: none"> Obtains expressions $d\theta = \frac{2dt}{1+t^2}$, $\cos\theta = \frac{1-t^2}{1+t^2}$ 	1

Question 2 (a) (i)*Outcomes assessed: E3***MARKING GUIDELINES**

Criteria	Marks
• Gives correct answer	1

Question 2 (a) (ii)*Outcomes assessed: E3***MARKING GUIDELINES**

Criteria	Marks
• Gives correct answer	1

Question 2 (b)*Outcomes assessed: E3***MARKING GUIDELINES**

Criteria	Marks
• Shades strip between 0 and 2 • Sketches circle centred at $(1, -1)$, radius 2 units • Restricts strip to inside circle	3
• Two of above	2
• One of above	1

Question 2 (c) (i)*Outcomes assessed: E3, E4, E9***MARKING GUIDELINES**

Criteria	Marks
• Gives appropriate explanation	1

Question 2 (c) (ii)*Outcomes assessed: E3, E4***MARKING GUIDELINES**

Criteria	Marks
• Gives correct answer	2
• Identifies either $(z + 4)$ or $(z^2 - 4z + 5)$ OR • Finds third root	1

Question 2 (d)*Outcomes assessed: HE2, E3***MARKING GUIDELINES**

Criteria	Marks
• Includes test for $n = 1$ • Uses induction assumption • Applies correct trigonometric identity	3
• Two of above	2
• One of above	1

Question 2 (e) (i)*Outcomes assessed: E3***MARKING GUIDELINES**

Criteria	Marks
• Gives correct answer	1

Question 2 (e) (ii)
Outcomes assessed: E2, E3
MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Multiplies by correct conjugate and simplifies correctly 	2
<ul style="list-style-type: none"> Multiplies by correct conjugate, but does not use $\cos^2 \theta + \sin^2 \theta = 1$ to simplify correctly OR	1
<ul style="list-style-type: none"> Multiplies by incorrect conjugate and uses $\sin^2 \theta + \cos^2 \theta = 1$ to simplify denominator 	

Question 2 (e) (iii)
Outcomes assessed: E3
MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Gives correct answer consistent with working in (ii) 	1

Question 3 (a) (i)
Outcomes assessed: E6
MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Draws reasonable graph (should show asymptotes and turning point) 	2
<ul style="list-style-type: none"> Makes no more than one significant error 	1

Question 3 (a) (ii)
Outcomes assessed: E6
MARKING GUIDELINES

Criteria	Marks
Draws reasonable graph, showing the following features: <ul style="list-style-type: none"> the isolated point at the origin the two branches meeting at (2, 0) 	2
<ul style="list-style-type: none"> Misses isolated point at the origin, but otherwise correct OR	1
<ul style="list-style-type: none"> Omits one of the branches meeting at (2, 0), but otherwise correct (including isolated point) 	

Question 3 (a) (iii)
Outcomes assessed: E6
MARKING GUIDELINES

Criteria	Marks
Draws correct graph, showing: <ul style="list-style-type: none"> – symmetry about $x = 0$ – cusps – turning points 	2
Draws correct graph for $x \geq 0$ OR <ul style="list-style-type: none"> • Incorrectly shows corners as smooth, but otherwise correct OR <ul style="list-style-type: none"> • Shows symmetry about $x = 0$ 	1

Question 3 (a) (iv)
Outcomes assessed: E6
MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> • Draws correct graph, showing graph cuts x-axis at $x = 3$ and asymptote 	2
<ul style="list-style-type: none"> • Shows domain of function implicitly on sketch 	1

Question 3 (b) (i)
Outcomes assessed: E3, E4
MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> • Correctly obtains equation $y - \frac{c}{p} = \frac{-c^2}{c^2 p^2}(x - cp)$ 	2
<ul style="list-style-type: none"> • Obtains gradient $\frac{-c^2}{c^2 p^2}$ at P 	1

Question 3 (b) (ii)
Outcomes assessed: E3, E4
MARKING GUIDELINES

Criteria	Marks
• Correctly demonstrates T is given point	2
• Attempts to solve $x + p^2y = 2cp$ $x + q^2y = 2cq$	1

Question 3 (b) (iii)
Outcomes assessed: E3, E4
MARKING GUIDELINES

Criteria	Marks
• Obtains $q = 2p$	3
• Correctly uses their expression for T to show the locus is a hyperbola	
• Correctly finds the eccentricity of their hyperbola	
• Two of the above	2
• One of the above	1

Question 4 (a) (i)
Outcomes assessed: P4
MARKING GUIDELINES

Criteria	Marks
• Finds correct coordinate	1

Question 4 (a) (ii)
Outcomes assessed: E7
MARKING GUIDELINES

Criteria	Marks
• Obtains correct integrand with limits consistent with (i)	3
• Obtains correct integrand	2
OR	
• Gives correct limits with simple mistake in integrand	1
• Clearly indicates understanding of method of cylindrical shells	

Question 4 (a) (iii)*Outcomes assessed: E8***MARKING GUIDELINES**

Criteria	Marks
• Obtains correct answer	2
• Correctly obtains indefinite integral, but fails to deal correctly with limits OR • Makes one error in otherwise correct working	1

Question 4 (b) (i)*Outcomes assessed: E2, E9***MARKING GUIDELINES**

Criteria	Marks
• Gives correct reasoning to show result	2
• Gives first significant step in correct argument eg showing $DSAR$ in cyclic	1

Question 4 (b) (ii)*Outcomes assessed: E2, E9***MARKING GUIDELINES**

Criteria	Marks
• Writes correct argument showing $\angle DST = \pi - \angle DCT$	2
• Gives first significant step in correct argument eg showing $DSTC$ is cyclic	1

Question 4 (b) (iii)*Outcomes assessed: E2, E9***MARKING GUIDELINES**

Criteria	Marks
• Provides any correct argument that R, S, T are collinear	2
• Gives first significant step in correct argument eg showing $\angle DCT + \angle DAB = \pi$	1

Question 4 (c) (i)*Outcomes assessed: HE3***MARKING GUIDELINES**

Criteria	Marks
• Gives correct answer	1

Question 4 (c) (ii)*Outcomes assessed: HE3***MARKING GUIDELINES**

Criteria	Marks
• Gives correct answer	2
• Recognises that there are 6 ways of ordering 3 numbers	1

Question 5 (a)*Outcomes assessed: E4***MARKING GUIDELINES**

Criteria	Marks
• Obtains correct answer	2
• Uses a correct method but obtains incorrect answer	1

Question 5 (b) (i)*Outcomes assessed: E4***MARKING GUIDELINES**

Criteria	Marks
• Finds correct equation	2
• Uses a correct method but obtains incorrect answer	1

Question 5 (b) (ii)*Outcomes assessed: E4***MARKING GUIDELINES**

Criteria	Marks
• Finds correct equation	2
• Uses a correct method but obtains incorrect answer	1

Question 5 (b) (iii)*Outcomes assessed: E4***MARKING GUIDELINES**

Criteria	Marks
• Finds correct value	2
• Uses a correct method but obtains incorrect answer OR correctly evaluates one of $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$	1

Question 5 (c) (i)*Outcomes assessed: E3, E4***MARKING GUIDELINES**

Criteria	Marks
<ul style="list-style-type: none">• Gives correct gradient for tangent• Correctly substitutes coordinates of P into point-gradient form of equation of tangent (or equivalent)	2
<ul style="list-style-type: none">• Obtains correct derivative	1

Question 5 (c) (ii)*Outcomes assessed: E3, E4***MARKING GUIDELINES**

Criteria	Marks
<ul style="list-style-type: none">• Correctly derives equation	2
<ul style="list-style-type: none">• Obtains conditions that T lies on tangents at P and Q	1

Question 5 (c) (iii)*Outcomes assessed: E3, E4***MARKING GUIDELINES**

Criteria	Marks
<ul style="list-style-type: none">• Provides a correct argument to prove TS and SR are perpendicular	3
<ul style="list-style-type: none">• Finds the coordinates of R, OR the gradient of TS, and indicates the product of the gradients is -1	2
<ul style="list-style-type: none">• Gives coordinates of S OR equation of directrix OR $b^2 = a^2(1 - e^2)$	1

Question 6 (a) (i)*Outcomes assessed: E5***MARKING GUIDELINES**

Criteria	Marks
<ul style="list-style-type: none">• Provides correct demonstration	1

Question 6 (a) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Attempts to resolve forces vertically and horizontally Includes $m\omega^2$ in equation for horizontal forces Provides any correct expression for $T - N$ 	3
<ul style="list-style-type: none"> Two of above 	2
<ul style="list-style-type: none"> One of above 	1

Question 6 (a) (iii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Substitutes $N = 0$ into equations of (ii) Obtains correct expression for ω or ω^2 	2
<ul style="list-style-type: none"> One of above 	1

Question 6 (b) (i)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Provides correct demonstration 	1

Question 6 (b) (ii)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Uses $\tan^2 \theta + 1 = \sec^2 \theta$ Substitutes $u = \tan \theta$, $du = \sec^2 \theta d\theta$ (or equivalent) Obtains $\int_0^1 u^{n-2} du = \frac{1}{n-1}$ 	3
<ul style="list-style-type: none"> Two of above 	2
<ul style="list-style-type: none"> One of above 	1

Question 6 (b) (iii)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Provides correct explanation of $I_{n+1} < I_n$ Correctly deduces $I_n < \frac{1}{2(n-1)}$ Correctly deduces $I_n > \frac{1}{2(n+1)}$ 	3
<ul style="list-style-type: none"> Two of above 	2
<ul style="list-style-type: none"> One of above 	1

Question 6 (b) (iv)

Outcomes assessed: E8, E9

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Correctly demonstrates result 	2
<ul style="list-style-type: none"> Obtains $I_5 = \frac{1}{2} \ln 2 - \frac{1}{4}$ OR <ul style="list-style-type: none"> Obtains $\frac{1}{12} < I_5 < \frac{1}{8}$ 	1

Question 7 (a) (i)

Outcomes assessed: HE5

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Uses $V = Ay$ 	1

Question 7 (a) (ii)

Outcomes assessed: E2, E5

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Provides correct derivation 	4
<ul style="list-style-type: none"> Provides unfinished correct derivation or derivation with a single mistake 	3
<ul style="list-style-type: none"> Provides derivation of relationship between t and y showing constants 	2
<ul style="list-style-type: none"> Separates variables 	1

Question 7 (a) (iii)

Outcomes assessed: E2

MARKING GUIDELINES

Criteria	Marks
• Obtains correct answer	2
• Substitutes either $\frac{y}{y_0} = \frac{1}{2}$ or $t = 10$ with incorrect answer	1

Question 7 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Provides correct explanation	2
• Provides partial explanation that shows understanding of vector addition	1

Question 7 (b) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Provides correct argument	2
• Shows $\angle P_0OP_1 = \angle P_0P_2P_1$	1

Question 7 (b) (iii)

Outcomes assessed: E2, E3, E9

MARKING GUIDELINES

Criteria	Marks
• Provides correct argument	2
• Shows that $P_0P_1P_2P_3$ is cyclic	1

Question 7 (b) (iv)

Outcomes assessed: E2, E3, E9

MARKING GUIDELINES

Criteria	Marks
• Provides correct argument	2
• Shows that vertices O, P_0, P_1, P_2, P_3 form a regular pentagon	1

Question 8 (a) (i)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Puts $n = 2m + 1$ into de Moivre's theorem and indicates the need to take the imaginary part	2
• Gives binomial expansion of $(\cos \theta + i \sin \theta)^{2m+1}$	
• One of above	1

Question 8 (a) (ii)

Outcomes assessed: E4, E9

MARKING GUIDELINES

Criteria	Marks
• Puts $x = \cot^2 \theta$ to obtain $p(x)$ from (i)	3
• Solves $\sin(2m + 1)\theta = 0, \sin \theta \neq 0$	
• Explains why $k = 1, 2, \dots, m$ gives solution	
• Two of above	2
• One of above	1

Question 8 (a) (iii)

Outcomes assessed: E2

MARKING GUIDELINES

Criteria	Marks
• Indicates LHS is sum of roots	2
• Shows RHS is $\binom{2m+1}{3} // \binom{2m+1}{1}$	
• One of above	1

Question 8 (a) (iv)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Applies $\cot \theta < \frac{1}{\theta}$ in (iii) Rearranges $\frac{m(2m-1)}{3} < \frac{(2m+1)^2}{\pi^2} \left[\frac{1}{1} + \frac{1}{2^2} + \dots + \frac{1}{m^2} \right]$ 	2
<ul style="list-style-type: none"> One of above 	1

Question 8 (b) (i)

Outcomes assessed: E2, E7

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Provides diagram or equivalent relating KL to $\triangle ABE$ Provides reasons for $KL = a - x$ Uses correct method to find LM Obtains $LM = a + x$ AND area = $a^2 - x^2$ 	4
<ul style="list-style-type: none"> Three of above 	3
<ul style="list-style-type: none"> Two of above 	2
<ul style="list-style-type: none"> One of above 	1

Question 8 (b) (ii)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
<ul style="list-style-type: none"> Obtains correct answer, given their answer to (i), provided the answer is positive 	2
<ul style="list-style-type: none"> Obtains correct expression for the volume as an integral 	1