

2002 HSC Mathematics Extension 2 Marking Guidelines

Question 1 (a)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Correctly evaluates integral, possibly omitting the constant of integration	2
• Obtains a relation equivalent to $du = \sec x \tan x dx$	1
OR	
• Obtains the expression $\int u^2 du$	

Question 1 (b)

Outcomes assessed: E8

Criteria	Marks
• Correctly evaluates integral by completing the square, possibly omitting the constant of integration	2
• Completes the square correctly (ie $x^2 + 2x + 2 = (x+1)^2 + 1$)	1



Question 1 (c)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correctly evaluates integral, possibly omitting the constant of integration	3
• Obtains result $\frac{3}{4}\ln(x+3) + \frac{1}{4}\ln(x-1)(+c)$	
• Obtains partial fraction expression $\frac{3}{4}\left(\frac{1}{x+3}\right) + \frac{1}{4}\left(\frac{1}{x-1}\right)$	2
• Gives partial fraction format $\frac{A}{x+3} + \frac{B}{x-1}$	1

Question 1 (d)

Outcomes assessed: E8

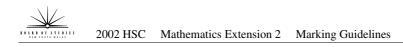
MARKING GUIDELINES

Criteria	Marks
Obtains correct answer	4
• Correctly integrates by parts twice, using correct choice for <i>u</i> , <i>dv</i> but fails to obtain final result	3
Correctly integrates by parts once, handling limits correctly	2
• Integrates by parts once. Gets correct result for integral except for a change of sign or failure to handle limits of integration	1

Question 1 (e)

Outcomes assessed: HE6

Criteria	Marks
Obtains correct result	4
Obtains correct primitive and correct limits of integration	3
• Obtains expression $\int_0^1 \frac{2dt}{t^3+3}$ or equivalent (in terms of <i>t</i>) or obtains correct primitive but with incorrect (or no) limits of integration	2
• Obtains expressions $d\theta = \frac{2dt}{1+t^2}, \cos\theta = \frac{1-t^2}{1+t^2}$	1



Question 2 (a) (i)

Outcomes assessed: E3

MARKING GUIDELINES	
Criteria	Marks
Gives correct answer	1

Question 2 (a) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
Gives correct answer	1

Question 2 (b)

Outcomes assessed: E3

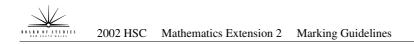
MARKING GUIDELINES

Criteria	Marks
• Shades strip between 0 and 2	3
• Sketches circle centred at (1, -1), radius 2 units	
Restricts strip to inside circle	
Two of above	2
One of above	1

Question 2 (c) (i)

Outcomes assessed: E3, E4, E9

Criteria	Marks
Gives appropriate explanation	1



Question 2 (c) (ii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
Gives correct answer	2
• Identifies either $(z + 4)$ or $(z^2 - 4z + 5)$	1
OR	
• Finds third root	

Question 2 (d)

Outcomes assessed: HE2, E3

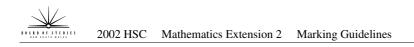
MARKING GUIDELINES

Criteria	Marks
• Includes test for $n = 1$	3
Uses induction assumption	
Applies correct trigonometric identity	
Two of above	2
One of above	1

Question 2 (e) (i)

Outcomes assessed: E3

Criteria	Marks
Gives correct answer	1



Question 2 (e) (ii)

Outcomes assessed: E2, E3

MARKING GUIDELINES

Criteria	Marks
Multiplies by correct conjugate and simplifies correctly	2
• Multiplies by correct conjugate, but does not use $\cos^2 \theta + \sin^2 \theta = 1$ to simplify correctly	1
OR	
• Multiplies by incorrect conjugate and uses $\sin^2 \theta + \cos^2 \theta = 1$ to simplify denominator	

Question 2 (e) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

	Criteria	Marks
•	Gives correct answer consistent with working in (ii)	1

Question 3 (a) (i)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Draws reasonable graph (should show asymptotes and turning point)	2
Makes no more than one significant error	1

Question 3 (a) (ii)

Outcomes assessed: E6

Criteria	Marks
Draws reasonable graph, showing the following features:	2
 the isolated point at the origin 	
- the two branches meeting at (2, 0)	
Misses isolated point at the origin, but otherwise correct	1
OR	
• Omits one of the branches meeting at (2, 0), but otherwise correct (including isolated point)	



Question 3 (a) (iii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
Draws correct graph, showing:	2
- symmetry about $x = 0$	
– cusps	
 turning points 	
Draws correct graph for $x \ge 0$	1
OR	
• Incorrectly shows corners as smooth, but otherwise correct	
OR	
• Shows symmetry about $x = 0$	

Question 3 (a) (iv)

Outcomes assessed: E6

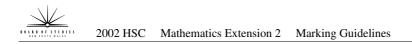
MARKING GUIDELINES

Criteria	Marks
• Draws correct graph, showing graph cuts x -axis at $x = 3$ and asymptote	2
Shows domain of function implicitly on sketch	1

Question 3 (b) (i)

Outcomes assessed: E3, E4

Criteria	Marks
• Correctly obtains equation $y - \frac{c}{p} = \frac{-c^2}{c^2 p^2} (x - cp)$	2
• Obtains gradient $\frac{-c^2}{c^2 p^2}$ at <i>P</i>	1



Question 3 (b) (ii)

Outcomes assessed: E3, E4

Criteria	Marks
• Correctly demonstrates <i>T</i> is given point	2
Attempts to solve	1
$x + p^2 y = 2cp$	
$x + q^2 y = 2cq$	

Question 3 (b) (iii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Obtains $q = 2p$	3
• Correctly uses their expression for <i>T</i> to show the locus is a hyperbola	
• Correctly finds the eccentricity of their hyperbola	
Two of the above	2
One of the above	1

Question 4 (a) (i)

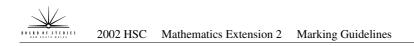
Outcomes assessed: P4

MARKING GUIDELINES Criteria Marks • Finds correct coordinate 1

Question 4 (a) (ii)

Outcomes assessed: E7

Criteria	Marks
Obtains correct integrand with limits consistent with (i)	3
Obtains correct integrand	2
OR	
• Gives correct limits with simple mistake in integrand	
Clearly indicates understanding of method of cylindrical shells	1



Question 4 (a) (iii)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
Obtains correct answer	2
• Correctly obtains indefinite integral, but fails to deal correctly with limits	1
OR	
Makes one error in otherwise correct working	

Question 4 (b) (i)

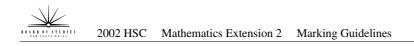
Outcomes assessed: E2, E9

	MARKING GUIDELINES		
	Criteria	Marks	
,	• Gives correct reasoning to show result	2	
(• Gives first significant step in correct argument eg showing DSAR in cyclic	1	

Question 4 (b) (ii)

Outcomes assessed: E2, E9

Criteria	Marks
• Writes correct argument showing $\angle DST = \pi - \angle DCT$	2
• Gives first significant step in correct argument eg showing <i>DSTC</i> is cyclic	1



Question 4 (b) (iii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Provides any correct argument that <i>R</i> , <i>S</i> , <i>T</i> are collinear	2
• Gives first significant step in correct argument eg showing $\angle DCT + \angle DAB = \pi$	1

Question 4 (c) (i)

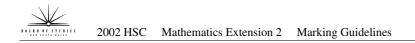
Outcomes assessed: HE3

MARKING GUIDELINES Criteria Marks • Gives correct answer 1

Question 4 (c) (ii)

Outcomes assessed: HE3

Criteria	Marks
Gives correct answer	2
• Recognises that there are 6 ways of ordering 3 numbers	1



Question 5 (a)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
Obtains correct answer	2
Uses a correct method but obtains incorrect answer	1

Question 5 (b) (i)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
Finds correct equation	2
Uses a correct method but obtains incorrect answer	1

Question 5 (b) (ii)

Outcomes assessed: E4

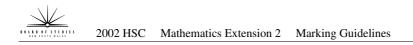
MARKING GUIDELINES

Criteria	Marks
Finds correct equation	2
Uses a correct method but obtains incorrect answer	1

Question 5 (b) (iii)

Outcomes assessed: E4

Criteria	Marks
Finds correct value	2
• Uses a correct method but obtains incorrect answer OR correctly evaluates one of $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$	1



Question 5 (c) (i)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
Gives correct gradient for tangent	2
• Correctly substitutes coordinates of <i>P</i> into point-gradient form of equation of tangent (or equivalent)	
Obtains correct derivative	1

Question 5 (c) (ii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
Correctly derives equation	2
• Obtains conditions that <i>T</i> lies on tangents at <i>P</i> and <i>Q</i>	1

Question 5 (c) (iii)

Outcomes assessed: E3, E4

MARKING GUIDELINES

Criteria	Marks
• Provides a correct argument to prove <i>TS</i> and <i>SR</i> are perpendicular	3
• Finds the coordinates of <i>R</i> , OR the gradient of <i>TS</i> , and indicates the product of the gradients is -1	2
• Gives coordinates of <i>S</i> OR equation of directrix OR $b^2 = a^2(1 - e^2)$	1

Question 6 (a) (i)

Outcomes assessed: E5

Criteria	Marks
Provides correct demonstration	1



Question 6 (a) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
Attempts to resolve forces vertically and horizontally	3
• Includes $mr\omega^2$ in equation for horizontal forces	
• Provides any correct expression for $T - N$	
Two of above	2
One of above	1

Question 6 (a) (iii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Substitutes $N = 0$ into equations of (ii)	2
• Obtains correct expression for ω or ω^2	
One of above	1

Question 6 (b) (i)

Outcomes assessed: E2, E8, E9

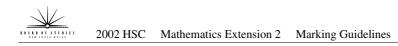
MARKING GUIDELINES

Criteria	Marks
Provides correct demonstration	1

Question 6 (b) (ii)

Outcomes assessed: E2, E8, E9

Criteria	Marks
• Uses $\tan^2 \theta + 1 = \sec^2 \theta$	3
• Substitutes $u = \tan \theta$, $du = \sec^2 \theta d\theta$ (or equivalent)	
• Obtains $\int_0^1 u^{n-2} du = \frac{1}{n-1}$	
Two of above	2
One of above	1



Question 6 (b) (iii)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES	
Criteria	Marks
• Provides correct explanation of $I_{n+1} < I_n$	3
• Correctly deduces $I_n < \frac{1}{2(n-1)}$	
• Correctly deduces $I_n > \frac{1}{2(n+1)}$	
Two of above	2
One of above	1

Question 6 (b) (iv)

Outcomes assessed: E8, E9

MARKING GUIDELINES

Criteria	Marks
Correctly demonstrates result	2
• Obtains $I_5 = \frac{1}{2} \ln 2 - \frac{1}{4}$	1
OR	
• Obtains $\frac{1}{12} < I_5 < \frac{1}{8}$	

Question 7 (a) (i)

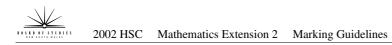
Outcomes assessed: HE5

MARKING GUIDELINES	
Criteria	Marks
• Uses $V = Ay$	1

Question 7 (a) (ii)

Outcomes assessed: E2, E5

Criteria	Marks
Provides correct derivation	4
• Provides unfinished correct derivation or derivation with a single mistake	3
• Provides derivation of relationship between <i>t</i> and <i>y</i> showing constants	2
Separates variables	1



Question 7 (a) (iii)

Outcomes assessed: E2

MARKING GUIDELINES	
Criteria	Marks
Obtains correct answer	2
• Substitutes either $\frac{y}{y_0} = \frac{1}{2}$ or $t = 10$ with incorrect answer	1

Question 7 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
Provides correct explanation	2
Provides partial explanation that shows understanding of vector addition	1

Question 7 (b) (ii)

Outcomes assessed: E3

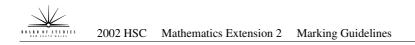
MARKING GUIDELINES

Criteria	Marks
Provides correct argument	2
• Shows $\angle P_0 OP_1 = \angle P_0 P_2 P_1$	1

Question 7 (b) (iii)

Outcomes assessed: E2, E3, E9

Criteria	Marks
Provides correct argument	2
• Shows that $P_0 P_1 P_2 P_3$ is cyclic	1



Question 7 (b) (iv)

Outcomes assessed: E2, E3, E9

MARKING GUIDELINES

Criteria	Marks
Provides correct argument	2
• Shows that vertices O, P_0, P_1, P_2, P_3 form a regular pentagon	1

Question 8 (a) (i)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Puts $n = 2m + 1$ into de Moivre's theorem and indicates the need to take the imaginary part	2
• Gives binomial expansion of $(\cos \theta + i \sin \theta)^{2m+1}$	
One of above	1

Question 8 (a) (ii)

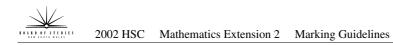
Outcomes assessed: E4, E9

	MARKING GUIDELINES		
	Criteria	Marks	
•	Puts $x = \cot^2 \theta$ to obtain $p(x)$ from (i)	3	
•	Solves $\sin(2m+1)\theta = 0$, $\sin\theta \neq 0$		
•	Explains why $k = 1, 2,, m$ gives solution		
•	Two of above	2	
•	One of above	1	

Question 8 (a) (iii)

Outcomes assessed: E2

Criteria	Marks
Indicates LHS is sum of roots	2
• Shows RHS is $\binom{2m+1}{3} / \binom{2m+1}{1}$	
One of above	1



Question 8 (a) (iv)

Outcomes assessed: E2, E9

MARKING GUIDELINES		
Criteria	Marks	
• Applies $\cot \theta < \frac{1}{\theta}$ in (iii)	2	
• Rearranges $\frac{m(2m-1)}{3} < \frac{(2m+1)^2}{\pi^2} \left[\frac{1}{1} + \frac{1}{2^2} + \dots + \frac{1}{m^2} \right]$		
One of above	1	

Question 8 (b) (i)

Outcomes assessed: E2, E7

MARKING GUIDELINES

Criteria	Marks
• Provides diagram or equivalent relating KL to ΔABE	4
• Provides reasons for $KL = a - x$	
• Uses correct method to find <i>LM</i>	
• Obtains $LM = a + x$ AND area $= a^2 - x^2$	
Three of above	3
Two of above	2
One of above	1

Question 8 (b) (ii)

Outcomes assessed: E7

Criteria	Marks
• Obtains correct answer, given their answer to (i), provided the answer is positive	2
Obtains correct expression for the volume as an integral	1