Tip: Click on the BOOKMARKS TAB to show / hide navigation

**Total marks – 120**
**Attempt Questions 1–8**
**All questions are of equal value**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

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**Marks**

**Question 1** (15 marks) Use a SEPARATE writing booklet.

(a) By using the substitution $u = \sec x$, or otherwise, find **2**

$$\int \sec^3 x \tan x \, dx .$$

(b) By completing the square, find $\displaystyle\int \frac{dx}{x^2 + 2x + 2}$. **2**

(c) Find $\displaystyle\int \frac{x\,dx}{(x+3)(x-1)}$. **3**

(d) By using two applications of integration by parts, evaluate **4**

$$\int_0^{\frac{\pi}{2}} e^x \cos x \, dx .$$

(e) Use the substitution $t = \tan\dfrac{\theta}{2}$ to find **4**

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{2 + \cos\theta} .$$

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Q 1

a) $\int \sec^3 x \tan x \, dx = \int \frac{u^2 (\sec x \tan x) \, du}{\sec x \tan x}$

let $u = \sec x$

$\frac{du}{dx} = \sec x \tan x.$

$= \frac{u^3}{3} + c$

$= \frac{\sec^3 x}{3} + c$

b) $\int \frac{dx}{x^2 + 2x + 2} = \int \frac{dx}{(x+1)^2 + 1}$

$= \tan^{-1}(x+1) + c$

$(x+1)^2$

$= x^2 + 2x + 1$

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c) $\displaystyle\int \frac{x\,dx}{(x+3)(x-1)}. = \int \frac{A}{x+3} + \frac{B}{x-1}\,dx.$

$x = A(x-1) + B(x+3).$

let $x = 1$

$1 = 0 + 4B.$

$B = \frac{1}{4}$

$x = A(x-1) + \frac{1}{4}(x+3).$

let $x = -3.$

$-3 = -4A + 0$

$A = \frac{3}{4}$

$\therefore \displaystyle\int \frac{x\,dx}{(x+3)(x-1)} = \int \frac{3}{4(x+3)}\,dx + \int \frac{1}{4(x-1)}\,dx$

$= \frac{3}{4}\ln|x+3| + \frac{1}{4}\ln|x-1| + C$

d)

$$\int_0^{\frac{\pi}{2}} e^x \cos x \, dx \qquad \text{let } u = \cos x$$

$$\underline{du} = -\sin x \, dx \, .$$

$$\int u \, dv = uv - \int v \, du$$

$$\text{let } dv = e^x$$

$$= \left[e^x \cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} e^x \sin x \, dx \qquad v = e^x$$

$$\int_0^{\frac{\pi}{2}} e^x \sin x \, dx \qquad \text{let } u = \sin x$$

$$du = \cos x \, dx \, .$$

$$= \left[e^x \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \cos x \, dx \, .$$

$$\text{let } dv = e^x$$

$$v = e^x$$

$$\therefore \int_0^{\frac{\pi}{2}} e^x \cos x \, dx = \left[e^x \cos x\right]_0^{\frac{\pi}{2}} + \left[e^x \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \cos x \, dx$$

$$2\int_0^{\frac{\pi}{2}} e^x \cos x \, dx = 0 - [1] + \left[e^{\frac{\pi}{2}}\right] - [0] .$$

$$2\int_0^{\frac{\pi}{2}} e^x \cos x \, dx = e^{\frac{\pi}{2}} - 1$$

$$\int_0^{\frac{\pi}{2}} e^x \cos x \, dx = \frac{e^{\frac{\pi}{2}} - 1}{2}$$

e) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{d\theta}{2+\cos\theta}$

let $t = \tan\frac{\theta}{2}$

$\frac{dt}{d\theta} = \frac{1}{2}\sec^2\frac{\theta}{2}$.

$d\theta = \frac{2dt}{1+t^2}$

when $\theta = \frac{\pi}{2}$
$t = 1$

when $\theta = 0$
$t = 0$

$$= \int_0^1 \frac{1}{2+\frac{1-t^2}{1+t^2}} \times \frac{2dt}{1+t^2}$$

$$= \int_0^1 \frac{1}{2+2t^2+1-t^2} \times 2dt.$$

$$= \int_0^1 \frac{1}{3+t^2} \times 2dt.$$

$$= \int_0^1 \frac{2dt}{3+t^2}$$

$$= \left[2\times\frac{1}{\sqrt3}\tan^{-1}\frac{t}{\sqrt3}\right]_0^1$$

$$= \left[\frac{2}{\sqrt3}\tan^{-1}\frac{1}{\sqrt3}\right] - \left[\frac{2}{\sqrt3}\tan^{-1}0\right]$$

$$= \left[\frac{2}{\sqrt3}\times\frac{\pi}{6}\right] - [0].$$

$$= \frac{2\pi}{6\sqrt3}.$$

$$= \frac{\pi}{3\sqrt3}$$