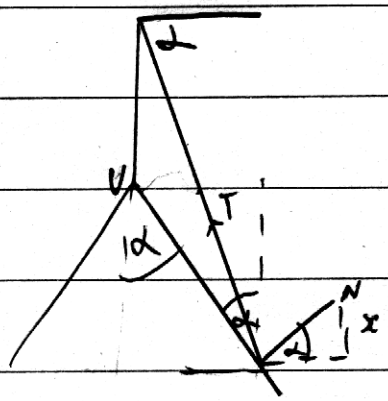
BOARD OF STUDIES
NEW SOUTH WALES

Question 6

a)



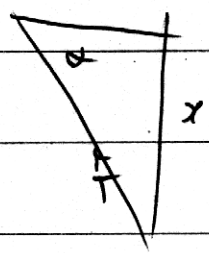
x is vertical component

as $\sin\alpha = x/N$

$x = N\sin\alpha$

∴ vertical component is $N\sin\alpha$

i)



$mg = T\cos\alpha + N\sin\alpha$

$mg = T + N(\sin\theta)$

$T + N = mg/\sin\alpha$

$T\cos\alpha - N\cos\alpha = mr\omega^2$

$T - N(\cos\alpha) = mr\omega^2$

$T - N = mr\omega^2/\cos\alpha$

$\cos\alpha = r/l$

$T - N = mr\omega^2/r/l$

$\therefore mr\omega^2 l$

$T - N = ml\omega^2$

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iii) $T + N = mg/\sin\alpha \qquad T - N = ml\omega^2$

at $N = 0$

$T = mg/\sin\alpha \qquad T = ml\omega^2$

$\dfrac{mg}{\sin\alpha} = ml\omega^2$

$\omega^2 = g/l\sin\alpha$

$\omega = \pm\sqrt{g/l\sin\alpha}$

$\omega = +\sqrt{g/l\sin\alpha}$ as ω is positive

b i) let $n = 1 \qquad I_1 = \int_0^{\pi/4} \tan\theta\, d\theta$

$= \int_0^{\pi/4} \sin\theta/\cos\theta\, d\theta$

$= -\left[\ln\cos\theta\right]_0^{\pi/4}$

$= -\ln\frac{1}{\sqrt{2}} - \ln 1$

$= -\left(\ln\left(\frac{1}{2}\right)^{1/2}\right)$

$= -\left(\frac{1}{2}\ln\frac{1}{2}\right)$

$= -\left[\frac{1}{2}\ln 1 - \frac{1}{2}\ln 2\right]$

$= \ln 2$

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ii) let $n = 2$

$LHS = I_2$

$\therefore$

iii) $I_n + I_{n-2} = \frac{1}{n-1}$

$I_n = \frac{1}{n-1} - I_{n-2}$

if $I_n < I_{n-2}$

iv) $I_5 = \frac{1}{4} - I_3$

$I_3 = 1 - I_1$

$I_5 = \frac{1}{4} - \left(1 - \frac{1}{2}\ln 2\right)$

$= -3/4 + \frac{1}{2}\ln 2$

$\frac{1}{2(5+1)} < I_5 < \frac{1}{2(5-1)}$

$1/12 < I_5 < 1/8$

$1/12 < \frac{-3}{4} + \frac{1}{2}\ln 2 < 1/8$

$\frac{5}{6} < \frac{1}{2}\ln 2 < \frac{3}{4}$

$\rightarrow \frac{2}{3} < \ln 2 < \frac{3}{4}$