Question 5 (15 marks) Use a SEPARATE writing booklet.
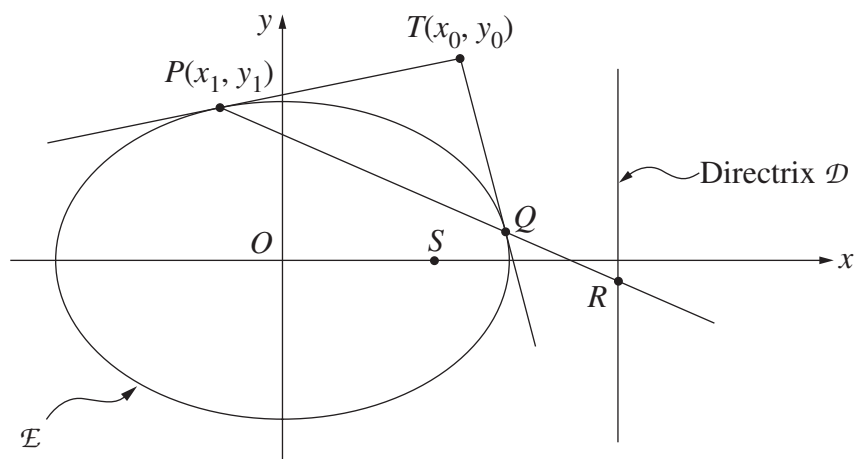
(a) The equation $4x^3 - 27x + k = 0$ has a double root. Find the possible values of k. **2**

(b) Let α, β, and γ be the roots of the equation $x^3 - 5x^2 + 5 = 0$.

   (i) Find a polynomial equation with integer coefficients whose roots are $\alpha - 1$, $\beta - 1$, and $\gamma - 1$. **2**

   (ii) Find a polynomial equation with integer coefficients whose roots are α^2, β^2, and γ^2. **2**

   (iii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. **2**

(c)

The ellipse $\mathcal{E}$ has equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, and focus S and directrix $\mathcal{D}$ as shown in the diagram. The point $T(x_0, y_0)$ lies outside the ellipse and is not on the x axis.

The chord of contact PQ from T intersects $\mathcal{D}$ at R, as shown in the diagram.

   (i) Show that the equation of the tangent to the ellipse at the point $P(x_1, y_1)$ is **2**

$$\frac{x_1 x}{a^2} + \frac{y_1 y}{b^2} = 1.$$

   (ii) Show that the equation of the chord of contact from T is **2**

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1.$$

   (iii) Show that TS is perpendicular to SR. **3**

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