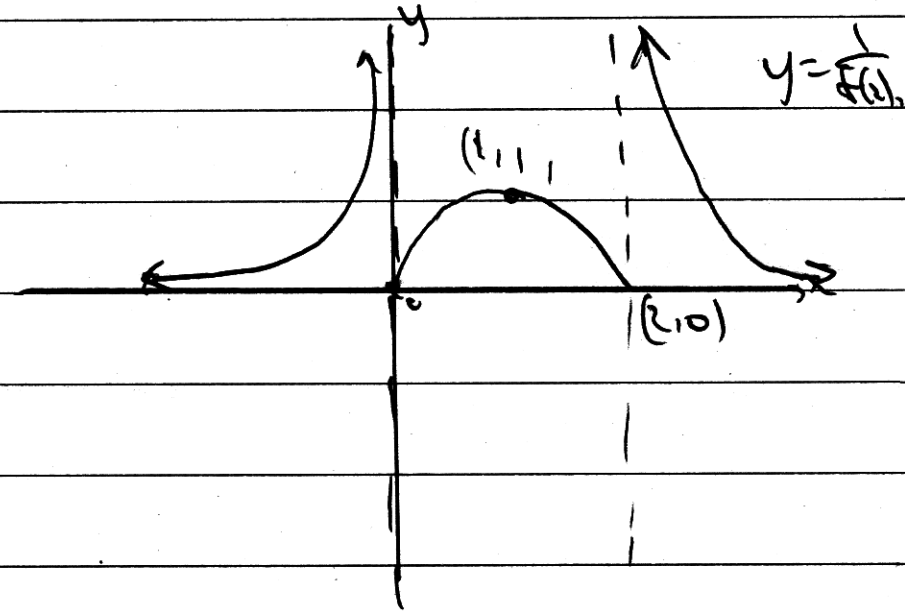
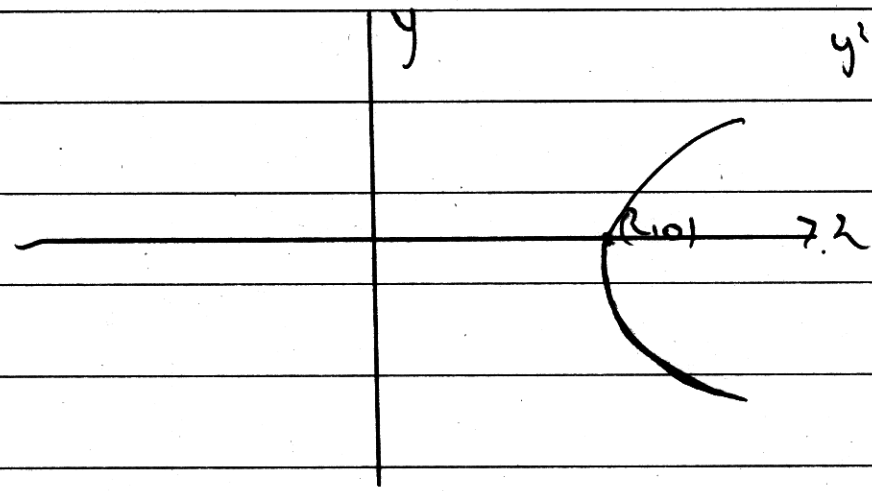
Question 3

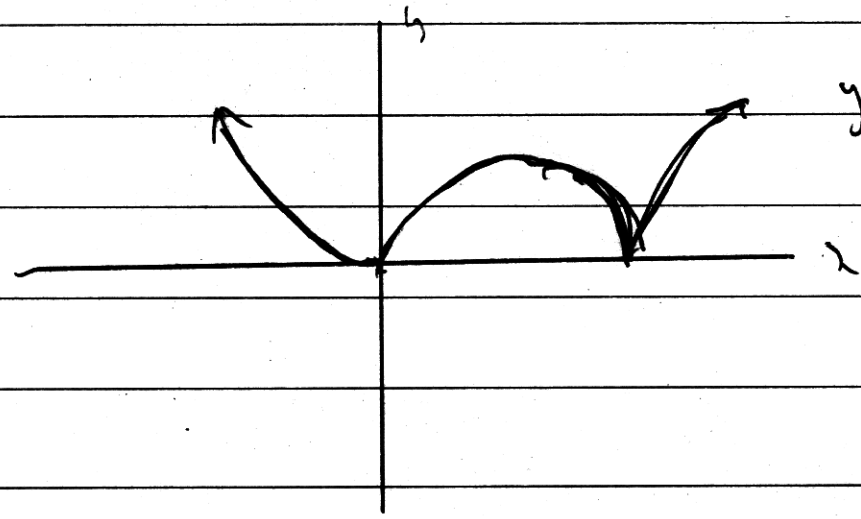
(a)(i)



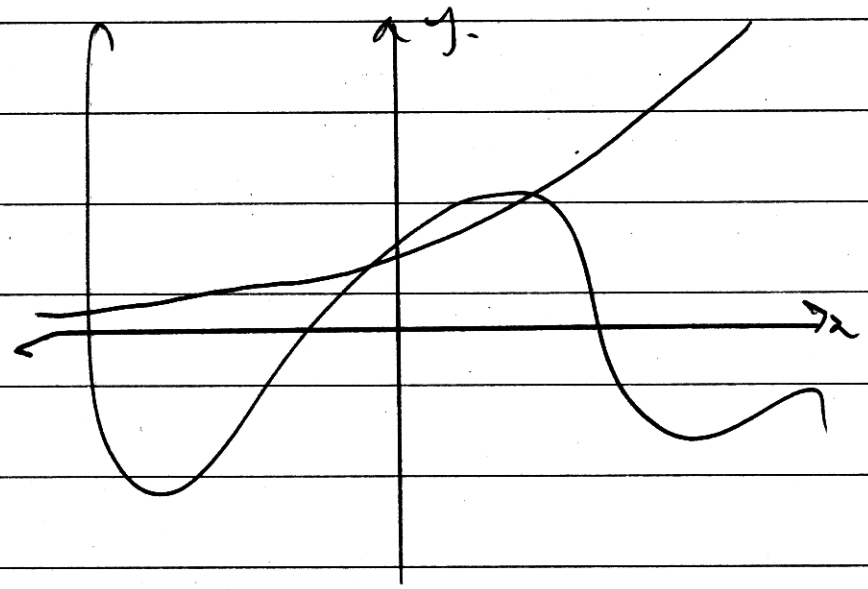
(ii)



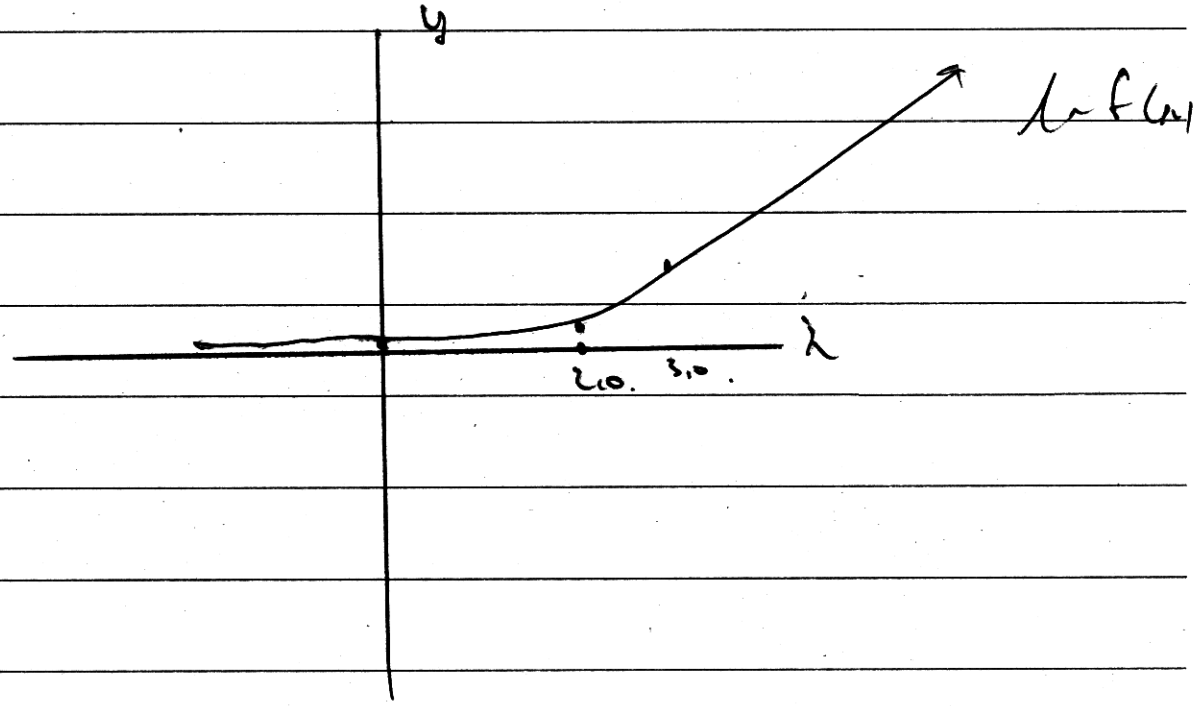
(iii)



(iv)



(iv)



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(b)(i) $xy = c^2$, $P(cp, \frac{c}{p})$, $Q(cq, \frac{c}{q})$.

$\frac{dy}{dx} = -c^2$

$y = \frac{c^2}{x}$

$dy = -\frac{c^2}{x}$

$x^{-1}\frac{1}{x}$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{2x}{a^2}dx = \frac{2y}{b^2}dy$

$\frac{dy}{dx} = \frac{\ldots}{\ldots}$

$\frac{1}{x^2}$

x^{-1}

$-\frac{x^{-1}}{x^{-1}}$

$\frac{dy}{dx} = 2c$

$y - \frac{c}{p} = 2c^2(x - cp)$

$py - c = 2c^2px - 2cp$

$py - c = 2cp(x - c)$

$\frac{py}{2cp} - \frac{c}{2cp}(x - c)$

$\frac{y}{2c}$

xyc^2

$dxdy = c^2$

$dx = dyc^2$

$y = \frac{c^2}{x}$

$dy = c^2(x)^{-1}$

$\frac{2c^2x^{-2}}{x}dx$

$\frac{1}{2}(c^2\frac{1}{x^2})$

$\frac{dy}{dx}$ $\frac{1}{2}(c^2 \cdot \frac{1}{x^2})$

$\frac{dy}{dx}$

$y - \frac{c}{p} = \frac{1}{2}\frac{c^2}{c^2p^2}(x - cp)$.

$2y - \frac{2c}{p} = \frac{1}{p^2}(x - cp)$.

$2yp^2 - 2cp = x - cp$

$x = 2yp^2 - 2cp + cp$.

$\cancel{x = 2yp^2 - cp}$

$x + p^2y = 2cp$.

$yp^2 - cp = \frac{2}{p^2}(x - cp$.

$yp^2 - cp = 2x - 2cp$

$2(x - 2cp$.

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(ii) tangent at p $x + p^2 y = 2cp$ — (1)

tangent at q $x + q^2 y = 2cq$ — (2)

$P(x = (2cp - p^2 y)$ sub into — (2)

$2cp - p^2 y + q^2 y = 2cq$

when $y = 0$, $x = 2cp$

$\frac{c}{q} - \frac{c}{p}$

(b)(i) $y = \frac{c^2}{x}$

$dy = -\frac{c^2}{x^2} dx$

$\frac{dy}{dx} = -\frac{c^2}{x^2}$

$y - \frac{c}{p} = -\frac{c^2}{cp}(x - cp)$

$y - \frac{c}{p} = -\frac{d}{p}(x - cp$

~~$py + c = cx - cp$~~

PTO

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NEW SOUTH WALES

$$y - \frac{c}{p} = -\frac{c^2}{c^2p^2}(x - cp)$$

$$-p^2y + pc = x - cp$$

$$x + p^2y = cp + cp$$

$$2pc = x + py^2 \quad \text{—— (1)}$$

(ii) $Q\left(cq, \frac{c}{q}\right)$ $\qquad \frac{dy}{dx} = \frac{c^2}{-x^2}$

$$y - \frac{c}{q} = -\frac{c^2}{c^2q^2}(x - cq)$$

$$-yq^2 + cq = (x - cq)$$

$$cq + cq = x + yq^2$$

$$2cq = x + qy^2 \quad \text{—— (2)}$$

$x = 2pc - py^2$ from (1). sub into (2)

$$2cq = 2pc - py^2 + qy^2$$

$$2c(q - p) = (qy^2 - py^2)$$

$$2c(q - p) = y^2(q - p)$$

$$2c = y^2 \qquad y = \sqrt{2c}$$