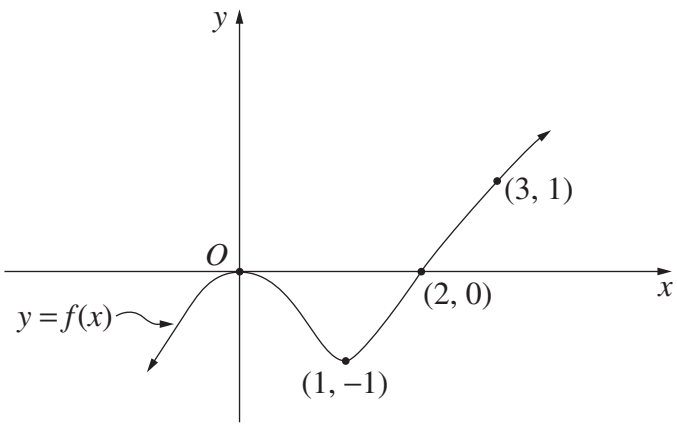
Marks

Question 3 (15 marks) Use a SEPARATE writing booklet.

(a)



The diagram shows the graph of $y = f(x)$.

Draw separate one-third page sketches of the graphs of the following:

(i) $y = \dfrac{1}{f(x)}$ **2**

(ii) $y^2 = f(x)$ **2**

(iii) $y = \left| f(|x|) \right|$ **2**

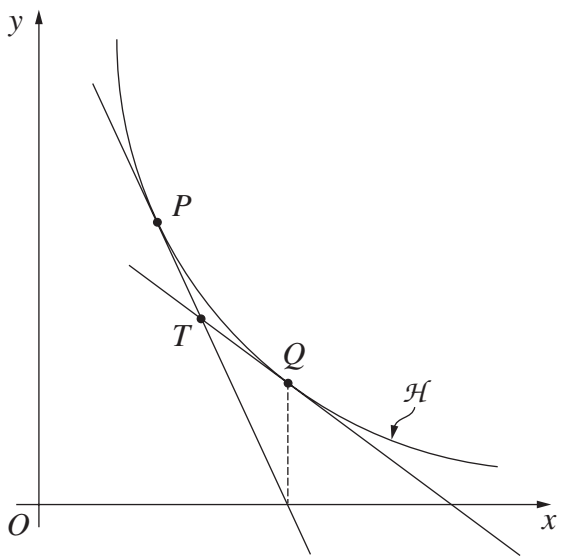
(iv) $y = \ln\big(f(x)\big)$. **2**

Question 3 continues on page 5

Marks

Question 3 (continued)

(b)



The distinct points $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ are on the same branch of the hyperbola $\mathcal{H}$ with equation $xy = c^2$. The tangents to $\mathcal{H}$ at P and Q meet at the point T.

(i) Show that the equation of the tangent at P is **2**

$$x + p^2y = 2cp.$$

(ii) Show that T is the point $\left(\frac{2cpq}{p+q}, \frac{2c}{p+q}\right)$. **2**

(iii) Suppose P and Q move so that the tangent at P intersects the x axis at $(cq, 0)$. **3**

Show that the locus of T is a hyperbola, and state its eccentricity.

End of Question 3