c) i) Since $2+i$ is a root, then $\overline{2+i}$ must be a root.

$\therefore$ $2-i$ is also a root of $P(z)$

ii) $\cancel{(2+i)(2-i)} =$

Since $2+i$ and $2-i$ are roots

$P[(2+i)(2-i)] = [(2+i)(2-i)]^3 + r[(2+i)(2-i)]^2$
$+ s[(2+i)(2-i)] + 20$

$0 = 125 + 25r + 5s + 20$

$= 5(25 + 5r + s + 4)$

d) $(\cos\theta - i\sin\theta)^n = \cos(n\theta) - i\sin(n\theta)$

when $n = 1$.

$LHS = \cos\theta - i\sin\theta$

$RHS = \cos\theta - i\sin\theta$

$\therefore LHS = RHS$.

Assume true for $n = k$.

$(\cos\theta - i\sin\theta)^k = \cos(k\theta) - i\sin(k\theta)$

a) i) $z\overline{w} = (1+2i)(1-i)$

$= 1 + i - 2i^2$

$= 3 + i$

ii) $\frac{1}{w} = \frac{1}{1+i} \cdot \frac{1-i}{1-i}$

$= \frac{1-i}{2}$

$= \frac{1}{2} - \frac{1}{2}i$

b) $0 \leq \text{Re}\, z \leq 2$ — (1)

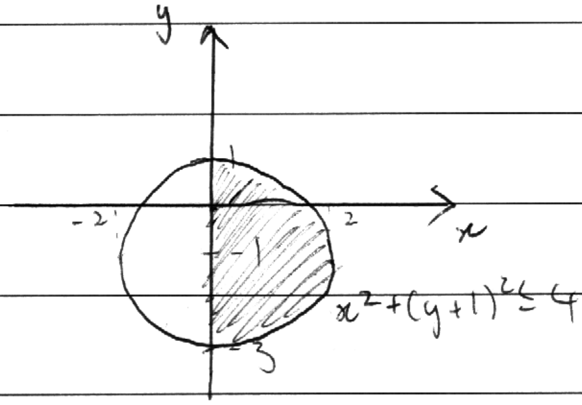
$|z-1+i| \leq 2$ — (2)

from (2). let $z = x+iy$

$|x+iy+i| \leq 2$

$\sqrt{x^2+(y+1)^2} \leq 2$

$x^2+(y+1)^2 \leq 4$



Prove true for $n=k+1$.

$(\cos\theta - i\sin\theta)^{k+1} = \cos[(k+1)\theta] - i\sin[(k+1)\theta]$

$LHS = (\cos\theta - i\sin\theta)^{k+1}$

$= (\cos\theta - i\sin\theta)^k(\cos\theta - i\sin\theta)$

$= [\cos(k\theta) - i\sin(k\theta)](\cos\theta - i\sin\theta)$

$= \cos(k\theta+\theta) - \cos k\theta . i\sin\theta - i\sin k\theta\cos\theta$

$+ i^2 \sin(k\theta+\theta)$

$= \cos[(k+1)\theta] - i\sin[(k+1)\theta]$

$= RHS$.

$\therefore$ The proposition is true for $n=k+1$ whenever it is true for $n=k$. As it is true for $n=1,2,$ etc, it is true for ~~n≥1~~ all integers of $n \geq 1$.

c) $z = 2(\cos\theta + i\sin\theta)$

$\overline{1-z} = 1+z$

$= 1 + 2(\cos\theta + i\sin\theta)$

$= 1 + 2\cos\theta + 2i\sin\theta$

BOARD OF STUDIES NEW SOUTH WALES

ii) $\frac{1}{1-z} = \frac{1}{1+2\cos\theta + \cancel{2}i\sin\theta}$

$= \frac{1+2\cos\theta + 2i\sin\theta}{(1+2\cos\theta)^2 + (2\sin\theta)^2}$

$= \frac{1+2\cos\theta + i\sin\theta}{1 + 4\cos\theta + 4\cos^2\theta + \sin^2\theta}$

$= \frac{1+2\cos\theta + i\sin\theta}{\cancel{1+4\cos\theta+4(\cos^2\theta+\sin^2\theta)}}$

$= \frac{1+2\cos\theta + i\sin\theta}{5+4\cos\theta}.$

Real part of $\frac{1}{1+z}$ is $\frac{1+2\cos\theta}{5+4\cos\theta}$

iii) Imaginary part is $\frac{\sin\theta}{5+4\cos\theta}.$