Marks

Question 2 (15 marks) Use a SEPARATE writing booklet.

(a) Let $z = 1 + 2i$ and $w = 1 + i$. Find, in the form $x + iy$,

(i) $z\overline{w}$ **1**

(ii) $\dfrac{1}{w}$. **1**

(b) On an Argand diagram, shade in the region where the inequalities **3**

$$0 \le \operatorname{Re} z \le 2 \text{ and } |z - 1 + i| \le 2$$

both hold.

(c) It is given that $2 + i$ is a root of

$$P(z) = z^3 + rz^2 + sz + 20,$$

where r and s are real numbers.

(i) State why $2 - i$ is also a root of $P(z)$. **1**

(ii) Factorise $P(z)$ over the real numbers. **2**

(d) Prove by induction that, for all integers $n \ge 1$, **3**

$$(\cos\theta - i\sin\theta)^n = \cos(n\theta) - i\sin(n\theta).$$

(e) Let $z = 2(\cos\theta + i\sin\theta)$.

(i) Find $\overline{1 - z}$. **1**

(ii) Show that the real part of $\dfrac{1}{1 - z}$ is $\dfrac{1 - 2\cos\theta}{5 - 4\cos\theta}$. **2**

(iii) Express the imaginary part of $\dfrac{1}{1 - z}$ in terms of θ. **1**