BOARD OF STUDIES
NEW SOUTH WALES

Q1

a). $\int \sec^3 x \tan x \, dx$ $\qquad u = \sec x$

$= \int \sec^2 x \, du$ $\qquad du = \sec x \tan x \, dx$

$= \int u^2 \, du = \left[\frac{u^3}{3}\right] = \left[\frac{\sec^3 x}{3}\right] + C$

b). $\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)(x+1)+1} = \int \frac{dx}{(x+1)^2+1}$

$\cancel{\tan^{-1} x}$ $= \tan^{-1}(x+1) + C$

c). $\int \frac{x\,dx}{(x+3)(x-1)}$

$= \frac{A}{(x+3)} + \frac{B}{(x-1)}$

$x = A(x-1) + B(x+3)$

$A + B = 1$ $\qquad A = 1 - B$

$0 = -A + 3B$

$0 = -1 + B + 3B$

$= -1 + 4B$

$1 = 4B$

$B = \frac{1}{4}$ $\qquad A = \frac{3}{4}$

$= \int \frac{3}{4(x+3)} dx + \int \frac{1}{(x-1)} dx$

$\cancel{= \int \frac{3}{12x+3}}$

$= \frac{3}{4}\int \frac{3 \times 4}{3 \times 4x+12} + \ln|x-1|$

$= \frac{3}{4}\ln|4x+12| + \ln|x-1| + C$

d). ~~$\int_0^{\frac{\pi}{2}} e^x \cos x \, dx$ $\qquad u = e^x \qquad dv = \cos x$~~

~~$du = e^x \qquad v = \sin x$~~

~~$= e^x \sin x - \int_0^{\frac{\pi}{2}} e^x \sin x \, dx$ $\qquad u = e^x \qquad dv = \sin x$~~

~~$= \left[e^x \sin x\right]_0^{\frac{\pi}{2}} + e^x \cos x + \int_0^{\frac{\pi}{2}} e^x \cos x$ $\qquad du = e^x \qquad v = -\cos x$~~

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d) $\int_0^{\frac{\pi}{2}} e^x \cos x \, dx$ $\quad u = e^x \quad dv = \cos x$

$= \left[e^x \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \sin x.$ $\quad du = e^x \quad v = \sin x$

$\cancel{= e^{\frac{\pi}{2}} - \left[e^x \sin x\right]_0^{\frac{\pi}{2}}}$ $\quad \cancel{u = e^x \sin x \quad dv = e^x} \quad \cancel{du = \cos x \quad v = e^x}$

$u = e^x \quad dv = \sin x$

$= e^{\frac{\pi}{2}} - \left(-\cos x e^x + \int_0^{\frac{\pi}{2}} e^x \cos x\right).$ $\quad du = e^x \quad v = -\cos x$

$= e^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \cos x$

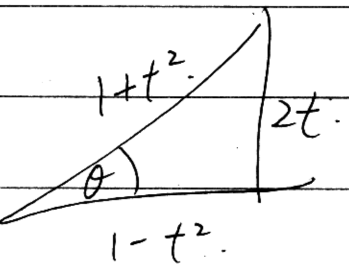
$\therefore 2\int_0^{\frac{\pi}{2}} e^x \cos x \, dx = e^{\frac{\pi}{2}}.$

$\int_0^{\frac{\pi}{2}} e^x \cos x \, dx = \frac{e^{\frac{\pi}{2}}}{2}.$

e) $\int_0^{\frac{\pi}{2}} \frac{d\theta}{2+\cos\theta}$ $\quad t = \tan\frac{\theta}{2}.$

$dt = \frac{1}{2}\sec^2\frac{\theta}{2}$

$= \int_0^1 \frac{1}{2 + \frac{1-t^2}{1+t^2}} \times \frac{2}{1+t^2}\, dt.$ $\quad = \frac{1+t^2}{2}\, d\theta.$



$= \int_0^1 \frac{2}{1+t^2+1-t^2}\, dt.$

$= \int_0^1 \frac{2}{2}\, dt.$

$\theta = \frac{\pi}{2} \quad t = 1.$

$= \left[t\right]_0^1$ $\quad \theta = 0 \quad t = 0$

$= 1.$