BOARD OF STUDIES
NEW SOUTH WALES

QUESTION 1

a) let $u = \sec x$ $\quad du = \sec x\, dx$ $\quad \sec^2 x \Rightarrow du = \tan x\, dx$

$\int \sec^3 x \tan x\, dx = \int u^3$

$= \int \sec^2 x \sec x \tan x\, dx \longrightarrow \int u^2 \tan x\, dx$

$= \int \sec x \sec x \cancel{\sec x \tan x\, dx} = \int \sec^2 x \tan x\, dx$

$\int \sec^2 x \tan x \sec x\, dx = \ln(\sec^2 x) + C$

$= \ln(\sec^2 x)$

$\int \sec^3 x \tan x\, dx$ $\quad$ let $u = \sec x$

$= \int \sec x \tan x \sec^2 x\, dx$

$= \cancel{\sec x} = \int \sec x \tan x \times \int \sec^2 x\, dx$

$= \int \sec x$

$= [u] \times \int u^2\, dx$

$= u + \frac{u^3}{3}$

$= \frac{3u + u^3}{3}$

$= \frac{3\sec x + \sec^3 x}{3}$

b) $\int \frac{dx}{x^2+2x+2}$

$x^2+2x+2=0$

$x^2+2x+4+2=4$

$(x^2+2)^2+2=0$

$= \int \frac{dx}{(x^2+2)^2+2}$

$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{(x^2+2)}{\sqrt{2}}$

c) i) $\int \frac{x\,dx}{(x+3)(x-1)}$

ie. $\frac{x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$

$x = A(x-1) + B(x+3)$

let $x=1$

$1 = 4B$

$B = 1/4$

let $x=-3$

$-3 = A(-4)$

$3 = 4A$

$4A = 3$

$A = 3/4$

ie. $\displaystyle\int \frac{x\,dx}{(x+3)(x-1)} = \int \frac{3}{4(x+3)}\,dx + \int \frac{1}{4(x-1)}\,dx$

$\displaystyle = \frac{3}{4}\int \frac{dx}{(x+3)} + \frac{1}{4}\int \frac{dx}{x-1}$

$= \frac{3}{4}\left(\ln(x+3)\right) + \frac{1}{4}\ln(x-1) + C$

$= \frac{1}{4}\left(3\ln(x+3) + \ln(x-1)\right) + C$

~~d~~

d) $\displaystyle\int_0^{\frac{\pi}{2}} e^x \cos x\,dx$ $\qquad v = e^x \quad u = \sin x$

$\qquad v' = e^x \quad u' = \cos x$

$\int vu' = uv - \int uv'$

$\displaystyle = \left[\sin x \cdot e^x\right]_0^{\frac{\pi}{2}} - \int \sin x\, e^x$ $\qquad v = e^x \quad u = -\cos x$

$\qquad v' = e^x \quad u' = \sin x$

$\displaystyle = \left[\sin\frac{\pi}{2}\cdot e^{\frac{\pi}{2}} - 0\right] - \left[-\cos x \cdot e^x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x\, e^x$

$= e^{\frac{\pi}{2}} + 0 + \left[\sin x \cdot e^x\right]_0^{\frac{\pi}{2}}$

$= e^{\frac{\pi}{2}} + e^{\frac{\pi}{2}} \Rightarrow = 2e^{\frac{\pi}{2}}$

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e) $t = \tan\frac{\theta}{2}$ $\qquad \sin\theta = \frac{2t}{1+t^2} \qquad \tan\theta = \frac{2t}{1-t^2}$

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{2+\cos\theta} \qquad \cos\theta = \frac{1-t^2}{1+t^2}$$

$t = \tan\frac{\theta}{2}$

$d\theta = \frac{1}{2}\tan\frac{\theta}{2}\, dt$

$$= \int_0^1 \frac{\frac{1}{2}\cdot\frac{2t}{1-t^2}/2}{2+\frac{1-t^2}{1+t^2}}$$

at $\theta = 0 \Rightarrow t = 0$

$\theta = \frac{\pi}{2} \Rightarrow t = 1$

$$= \int_0^1 \frac{\frac{2}{1-t^2}}{\frac{2(1+t^2)+1-t^2}{1+t^2}}$$

$$= \int_0^1 \frac{\frac{2}{1-t^2}}{\frac{2+2t^2+1-t^2}{1+t^2}}$$

$$= \int_0^1 \frac{2}{1-t^2} \times \frac{1+t^2}{3-t^2}$$

$$\int_0^1 \frac{2(1+t^2)}{(1-t^2)(3-t^2)}$$

$$= 2\int_0^1 \frac{1+t^2}{(1+t^2)(3-t^2)}$$

$$= 2\int_0^1 \frac{1}{(1+t^2)(3-t^2)} + \frac{t^2}{(1+t^2)(3-t^2)}$$

$$= \cancel{\int_0^1 \frac{2}{(1+t)}}$$